# THEORY OF ESTIMATION

- Estimation Of
  - Point,
  - Interval and
  - Sample Size.

# INTRODUCTION:

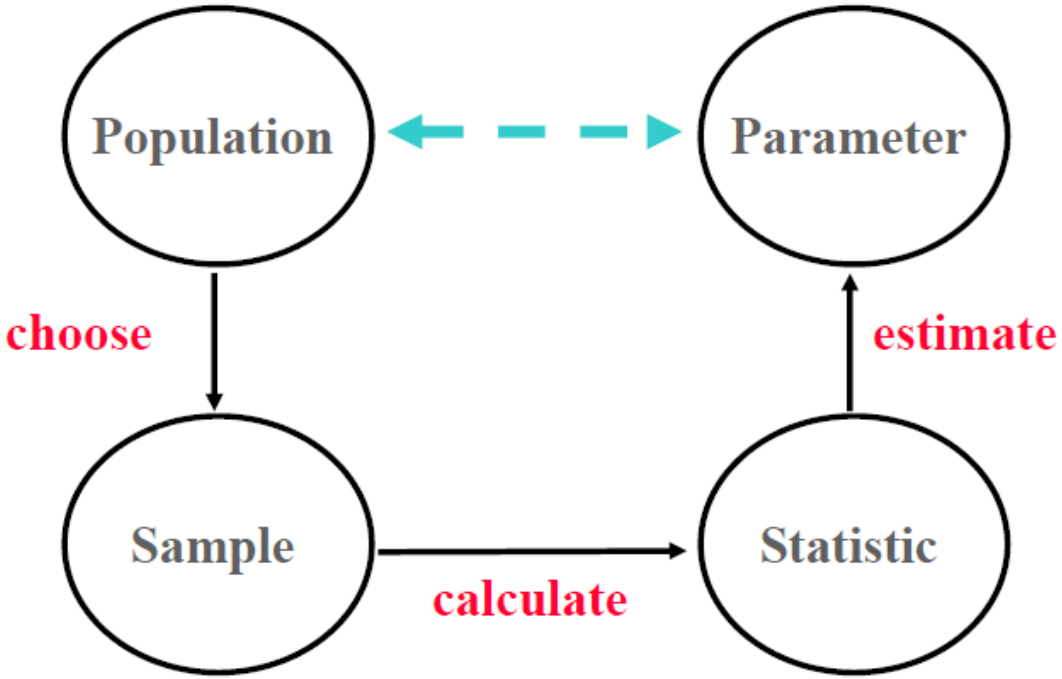
- Estimation Theory is a procedure of "guessing" properties of the population from which data are collected.

- i.e, The objective of estimation is to determine the ***approximate value*** of a population parameter on the basis of a sample statistic.

- An **estimator** is a rule, usually a formula, that tells you how to calculate the estimate based on the sample.

# PROPERTIES OF GOOD ESTIMATORS

- **Unbiased**: the average value of the estimator equals the parameter to be estimated.
- **Minimum variance**: of all the unbiased estimators, the best estimator has a sampling distribution with the smallest standard error



# TOPICS TO BE DISCUSSED:

- **Point Estimate:** A point estimate is a one-number summary of data
- **Interval Estimation:** Two numbers are calculated to create an interval within which the parameter is expected to lie..
- For example, suppose we want to estimate the mean summer income of a class of business students.
- Point Estimate:
  - For n=25 students,  is calculated to be 400 $/week.
- Interval Estimate:
  - An alternative statement is:
  - The mean income is ***between*** 380 and 420 $/week.

# Sample Size

- "Sample Size" - is the number of a population that will be evaluated as representing the entire population, and from which statistics will be derived.
- The sample size is an important feature of any empirical study in which the goal is to make inferences about a population from a sample.
- In practice, the sample size used in a study is determined based on the expense of data collection, and the need to have sufficient statistical power .

- The larger the sample, the closer we get to the population.
- Too large is unethical, because it's **wasteful**.
- Too small is unethical, because the outcome will be **indecisive**.
- If you get significance and you're wrong, it's a false-positive or **Type I statistical error**.
- If you get non-significance and you're wrong, it's a false negative or **Type II statistical error**.

## Factors That Influence Sample Size

- The "right" sample size for a particular application depends on many factors, including the following:
- **Cost considerations** (e.g., maximum budget, desire to minimize cost).
- **Administrative concerns** (e.g., complexity of the design, research deadlines).
- **Minimum acceptable level of precision**.
- **Confidence level.**
- **Variability within the population or subpopulation** (e.g., stratum, cluster) of interest.
- **Sampling method.**

Ex:

- In a survey sampling involving stratified sampling there would be different sample sizes for each population. In a census, data are collected on the entire population, hence the sample size is equal to the population size
- **Stratified sample size**
- With more complicated sampling techniques, such as stratified sampling, the sample can often be split up into sub-samples.
- Typically, if there are $k$ such sub-samples (from $k$ different strata) then each of them will have a sample size $n_i$, $i = 1, 2, ..., k$. These $n_i$ must conform to the rule that $n_1 + n_2 + ... + n_k = n$ (i.e. that the total sample size is given by the sum of

# ESTIMAION OF SAMPLE POINT:

- **A single number is calculated to estimate the parameter.**
- A point estimate is obtained by selecting a suitable statistic and computing its value from the given sample data. The selected statistic is called the point estimator of $\theta$. $\hat{\theta}$ $\hat{\theta}$ $\theta$
- A point estimate of an unknown parameter is a statistic that represents a “guess” at the value of .
- **Parameters**
  - In statistical inference, the term parameter is used to denote a quantity , say, that is a property of an unknown probability distribution.
  - Parameters are unknown, and one of the goals of statistical inference is to estimate them.

- Example (Machine breakdowns)
  - Estimating

    **P(machine breakdown due to operator misuse).**
- Some general Concepts of Point Estimation:
  - Unbiasedness.
  - Principle of Minimum Variance.
- Methods of Point Estimation:
  - Maximum Likelihood Estimation.
  - The Method of Moments.

# Point Estimator Of Population Mean

A point estimate of population mean $\mu$ is the sample mean

$$\bar{x} = \frac{\sum x_i}{n}$$

A sample of weights of 34 male freshman students was obtained.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 185 | 161 | 174 | 175 | 202 | 178 | 202 | 139 | 177 |
| 170 | 151 | 176 | 197 | 214 | 283 | 184 | 189 | 168 |
| 188 | 170 | 207 | 180 | 167 | 177 | 166 | 231 | 176 |
| 184 | 179 | 155 | 148 | 180 | 194 | 176 | | |

If one wanted to estimate the true mean of all male freshman students, you might use the sample mean as a point estimate for the true mean.

sample mean $= \bar{x} = 182.44$

# BIASED & UNBIASED

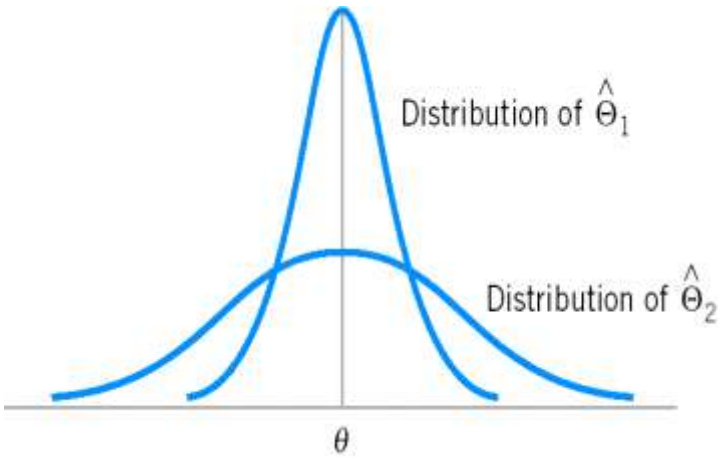
- A point estimate $\hat{\theta}$ for a parameter is said to be unbiased if $E(\hat{\theta}) = \theta$
- If this equality does not hold, $\hat{\theta}$ is said to be a **biased estimator of θ,** with $bias = E(\hat{\theta}) - \theta$

# Variance of a Point Estimator

If we consider all unbiased estimators of $\theta$, the one with the smallest variance is called the **minimum variance unbiased estimator** (MVUE).

If $X_1, X_2, \ldots, X_n$ is a random sample of size $n$ from a normal distribution with mean $\mu$ and variance $\sigma^2$, the sample mean $\overline{X}$ is the MVUE for $\mu$.

- The sampling distributions of two unbiased estimators.
- Of all the **unbiased** estimators, we prefer the estimator whose sampling distribution has the **smallest spread** or **variability**.



# INTERVAL ESTIMATES

- An Estimation of a population parameter given by two numbers between which the parameter may be called as an *internal estimation of the parameter*.
- Eg : If we say that a distance is 5.28 feet, we are giving a point estimate. If, on the other hand, we say that the distance is 5.28 ± 0.03 feet, i.e., the distance lies between 5.25 and 5.31 feet, we are giving an interval estimate.
- A statement of the error or precision of an estimate is often called *its reliability.*

# CONFIDENCE INTERVAL ESTIMATES OF POPULATION PARAMETERS

- Let μ*S and σS be the mean and standard deviation of the* sampling distribution of a statistic *S.*
- *Then, if the sampling distribution* of *S is approximately normal* we can expect to find *S lying in the* interval μ*S . σS to μS + σS, μS . 2σS to μS + 2σS or μS . 3σS to μS + 3σS* about 68.27%, 95.45%, and 99.73% of the time, respectively.
- *We can be con.dent of .nding μS in the intervals S. σS to S + σS, S . 2σS to S + 2σS, or S . 3σS to S + 3σS about 68.27%, 95.45%, and 99.73% of the time, respectively. Because of this, we call*

# CONFIDENCE LIMITS:

- The end numbers of these intervals (*S ± σS, S ± 2 σS, S ± 3 σS) are then called the 68.37%, 95.45%, and* 99.73% *Confidence Limits.*

# CONFIDENCE LEVEL :

- *S ± 1.96 σS and S ± 2.58 σS are 95% and 99% (or 0.95 and*0.99) confidence limits for μ*S. The percentage confidence is often called Confidence Level.*

# CRITICAL VALUE :

- The numbers 1.96, 2.58, etc., in the confidence limits are called *Critical Values, and are denoted by zC. From confidence* levels we can find critical values.

**Eg:**

we give values of *zC corresponding to various confi*dence levels used in practice. For confidence levels not presented in the table, the values of *zC can be found from the normal curve area*s under the Standard Normal Curve from 0 to *z.*

| **C L** | 99.7% | 99% | **98 %** | **96%** | **95.45 %** | **95%** | **90%** | **80%** | **68.27 %** |
|---|---|---|---|---|---|---|---|---|---|
| ***zC*** | 3.00 | 2.58 | 2.33 | 2.05 | 2.00 | 1.96 | 1.645 | 1.28 | 1.00 |

- In cases where a statistic has a sampling distribution that is different from the normal distribution, appropriate modifications to obtain confidence intervals have to be made.

## CONFIDENCE INTERVALS:

- Confidence Intervals for Means
- Confidence Intervals for Proposition
- Confidence Intervals for Differences and Sums.

# Confidence Intervals for Means :

- We shall see how to create confidence intervals for the mean of a population using two different cases.
- The first case shall be when we have a *Large Sample Size (N ≥ 30).*
- *The second case shall be when we* have a *Smaller Sample (N < 30).*
- *Then Underlying Population is normal.*

# Large Samples (*n* ≥ *30*) :

- If the statistic *S is the sample mean X*, then the 95% and 99% confidence limits for estimation of the population mean μ are given by $X \pm 1.96\,\sigma X$ *and* $X \pm 2.58\,\sigma X$, *respectively.*
- *The confidence* limits are given by $X \pm zc\,\sigma X$ *where zc, which depends on the particular* level of confidence desired.

$$\overline{X} \pm z_C \frac{\sigma}{n}$$

- In case sampling from an infinite population or if sampling is done with replacement from a finite population, and by

$$\overline{X} \pm z_C \frac{\sigma}{n} \sqrt{\frac{N-n}{N-1}}$$

- If sampling is done without replacement from a population of finite size *N.*
- The population standard deviation σis unknown, so that to obtain the above confidence limits, we use the estimator *Sˆ or S.*

# Small Samples (n < 30) and Population Normal :

• We use the *t distribution* to obtain confidence levels. For example, if *–t0.975 and t0.975 are the values of T for* which 2.5% of the area lies in each tail of the *t distribution, then a 95%* confidence interval for *T is given by*

$$-t_{0.975} < \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} < t_{0.975}$$

*from which we can see that μ can be estimated to lie in the interval with 95% confidence*

$$\bar{X} - t_{0.975}\frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{0.975}\frac{S}{\sqrt{n}}$$

- In general the confidence limits for population means are given by where the *tc values.*

$$\bar{X} \pm t_c \frac{\hat{S}}{\sqrt{n}}$$

•Sample size is very important! We construct different confidence intervals based on sample size, so make sure we know which procedure to use.

# Confidence Intervals for Proportions :

- The statistic *S is the proportion of "successes" in a sample* of size *n ≥ 30 drawn from a binomial population in which p is the proportion* of successes.
- Then the confidence limits for *p are given by P ± zc σP, where P denotes the proportion* of success in the sample of size *n. Using the values of σP obtained*, we see that the confidence limits for the population proportion are given by

$$P \pm z_c \sqrt{\frac{pq}{n}} = P \pm z_c \sqrt{\frac{p(1-p)}{n}}$$

- In case sampling from an infinite population or if sampling is with replacement from a finite population. Similarly, the confidence limits are if sampling is without replacement from a population of finite size *N.*

$$P \pm z_c \sqrt{\frac{pq}{n}} \sqrt{\frac{N-n}{N-1}}$$

# Confidence Intervals for Differences and Sums :

- If *S1 and S2 are two sample statistics with approximately normal sampling* distributions, confidence limits for the differences of the population parameters corresponding to *S1 and S2 are given by*

$$S_1 - S_2 \pm z_c \sigma_{S_1 - S_2} = S_1 - S_2 \pm z_c \sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2}$$

while confidence limits for the sum of the population parameters are given by provided that the samples are independent.

$$S_1 + S_2 \pm z_c \sigma_{S_1+S_2} = S_1 + S_2 \pm z_c \sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2}$$

- Confidence limits for the difference of two population means, in the case where the populations are infinite and have known standard deviations σ1, σ2, are given by

$$\bar{X}_1 - \bar{X}_2 \pm z_c \sigma_{\bar{X}_1 - \bar{X}_2} = \bar{X}_1 - \bar{X}_2 \pm z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- Where $\bar{X}_1, n_1$ and $\bar{X}_2, n_2$ *are the respective means and sizes of the two* samples drawn from the populations.
- Confidence limits for the difference of two population proportions, where the populations are infinite, are given by

$$P_1 - P_2 \pm z_c \sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}$$

- where *P1 and P2 are the two sample proportions and n1 and n2 are the* sizes of the two samples drawn from the populations.

## VARIANCE :

- The variance for the difference of means is the same as the varia f means.

$$\sigma^2_{x+y} = \sigma^2_{x-y}$$

# THANK YOU…